HADRON-HADRON INTERACTIONS IN THE CONSTITUENT QUARK MODEL: RESULTS AND EXTENSIONS

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Abstract

Hadronic interactions are discussed within the context of the constituent quark model. The "Quark Born Diagram" methodology is outlined, extensive applications to meson-meson and meson-baryon interactions are discussed, and general features of these interactions are highlighted. The second half of this document deals with shortcomings of the quark model approach and methods to overcome them. These include relativistic kinematics, unitarity, nonlocal potentials, coupled channel effects, and the chiral nature of the pion.

1 Ubiquitous Hadronic Interactions

A microscopic understanding hadron-hadron scattering remains an elusive goal of hadronic physics. This is unfortunate because the interactions of hadrons is important from a variety of perspectives. At the hadronic level, it provides vital insight into the dynamics of quarks and gluons. It is also relevant to the search for the quark gluon plasma, since hadronic interactions can mask putative signals for the QCD phase transition. Applications extend beyond hadronic physics: for example the search for CP violating phases in the final states of D of B decays will require correctly accounting for strong interaction final state phases[1]. Of course, hadron-hadron interactions are directly relevant to a longstanding goal of nuclear physics – deriving the nuclear force from QCD. This issue is not just of intellectual concern since it is important to be able to extend our understanding of internuclear forces to extreme conditions (of temperature and pressure) so that a variety of astrophysical and cosmological issues may be reliably examined.

Given the importance of the area, it is not surprising that a number of techniques have been developed to address the dynamics of hadron-hadron interactions. These roughly fall into two classes: those which treat hadrons as elementary fields and those which attempt to describe the interactions using

QCD as the starting point. Among the former are potential approaches[2] (Bonn, Paris, Argonne), relativistic hydrodynamic approaches[3], and a variety of effective field theories[4] (chiral perturbation theory, effective NN theory). Among the latter are lattice gauge theory[5], Schwinger-Dyson models[6], light cone field theory[7], a multigluon dipole interaction model[8], and constituent quark models.

Unfortunately, it is not possible to summarize the advantages and faults of all of these methods here. Rather, the remainder of this document focuses on attempts to describe hadronic interactions which are based on the nonrelativistic constituent quark model (CQM). The CQM is beyond a doubt the most widely used description of the static properties of hadrons – largely because it is able to describe hundreds of experimental data with a handful of parameters in a comparatively simple picture. Thus it is no surprise that attempts to describe hadronic interactions with the CQM have a long history [9]. Despite the well-known short comings of the CQM (some of which will be discussed below) the benefits are immediately apparent: the CQM may be applied to *any* hadronic interactions and the predictions *are essentially parameter-free*. By this I mean that the parameters of the CQM are very strongly fixed by comparison with static hadron properties – there is no wiggle room when computing interactions! Of course the extension of the CQM to dynamic properties of hadrons requires some extrapolation[1]; however, this should not form a barrier in itself. To paraphrase Feynman, "Trust your model and see how far it takes you."

2   Quark Born Diagrams

In the following we shall consider a CQM which includes Coulomb and linear central potentials, a spin-spin colour hyperfine interaction, and possibly spin-orbit and tensor interactions. It may be somewhat of a surprise that the Breit-Fermi interactions are included here since they are normally only required to achieve detailed agreement with spectroscopic and other static properties of the hadrons. However, as we shall see shortly, subleading (in v/c) interactions can dominate hadronic scattering!

A great deal of effort has been expended on variational[10] and resonating group approaches[11] to scattering in the CQM. Here I describe a simplified approach where one evaluates the T-matrix at Born order[12, 13], called the "Quark Born Diagram" (QBD) method. The T-matrix for meson-meson scat-

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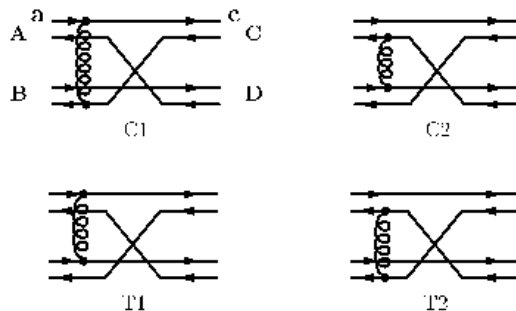

Fig. 1: Diagrams Contributing to Meson-Meson Scattering

2.1   Applications

The application of the Quark Born formalism requires some care. For example, if one is to restrict attention to the terms shown in Fig 1, then channels where strong quark annihilation effects are expected (such as $I = 0 \ \pi\pi$) should be avoided. Furthermore, if the predicted interactions are strong, the Born order results should be iterated (this is discussed below). The Quark Born Diagram method has been applied to a variety of hadronic reactions. These include I=2 $\pi\pi$ scattering[12, 15] (with surprising agreement considering the relativistic and chiral nature of the pion), I=3/2 πK scattering[16] (testing Bose symmetry breaking due to the strange quark mass), KN scattering[17] (demonstrating surprising agreement in the S-wave and a dramatic failure in

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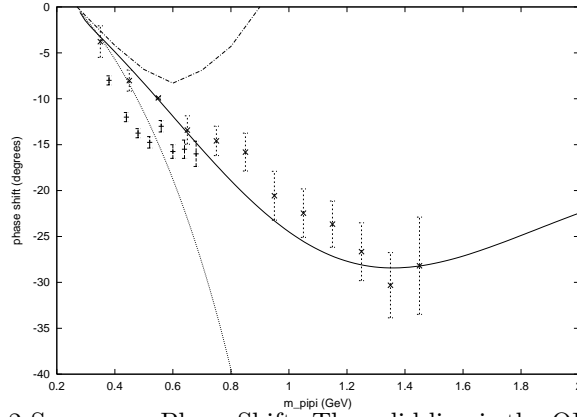

Fig. 2: The $I = 2$ S-wave $\pi\pi$ Phase Shift. The solid line is the QBD prediction[12], the dashed line is the one loop chiral prediction[23], while the lower dashed line is the tree order chiral result. Data are from [24].
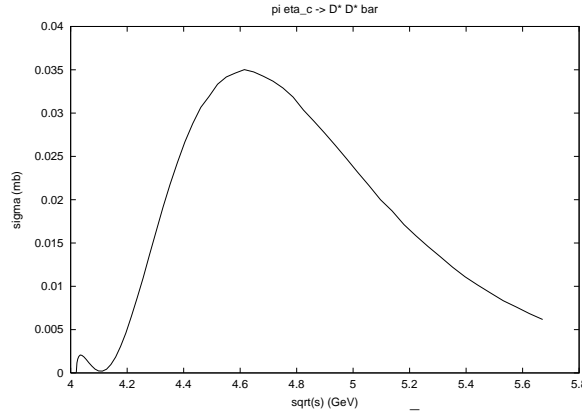

Fig. 3: Prediction of the $\pi\eta_c \to D^*\bar{D}^*$ Cross Section.

3   Limitations and Extensions of the CQM

As mentioned above, the CQM in general, and the QBD formalism in particular, suffers from several inadequacies. We summarize several of these and discuss methods for addressing them.

3.1   Unitarity, Relativistic Kinematics, and Nonlocality

Perhaps the simplest problem arises when the scattering is so strong that the tree level diagrams of the QBD method are inaccurate. This may be tested by comparing QBD predictions to more complete resonating group calculations (which iterate the scattering to all orders and can, in principle, include the effects of coupled channels, wavefunction distortion, etc). Such a comparison of $\rho\rho$ scattering in isospin 2 was made in Ref. [13] where the accuracy of the QBD results were explicitly demonstrated. Nevertheless, it is worth observing that strong interactions may be accounted for by extracting effective potentials for the process in question and iterating the potential in the appropriate Schrödinger equation. An example of this is shown in Fig 4, where the effective $\pi\pi$ interaction has been extracted from the QBD $\pi\pi$ T-matrix and iterated in the nonrelativistic Schrödinger equation. This is shown as a dashed line. Evidently, the agreement with data is ruined by this procedure (I am presenting the worst possible case – the procedure works very well in general). This is because the light mass of the pions (and the large invariant masses at which the formalism is being applied) requires relativistic kinematics to be employed (this has been used in the QBD prediction of Fig 2). Thus the nonrelativistic Schrödinger equation is inappropriate for iterating the effective $\pi\pi$ interaction.
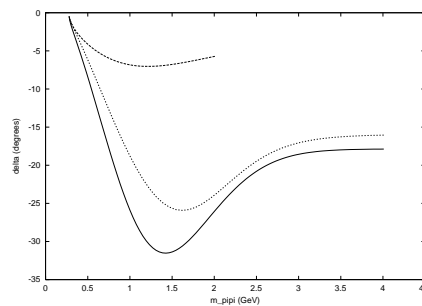

Fig. 4: I=2 $\pi\pi$ Scattering. The solid line is the QBD prediction, the dashed line is the nonrelativistic local prediction, while the dotted line is the nonlocal, relativistic prediction.

A related problem is the nonlocality of the $\pi\pi$ T-matrix (because composite particles are being scattered). Any extracted effective potential which is local

will induce unknown errors after being iterated. Fortunately, all three problems may be dealt with by choosing to solve for the T-matrix directly in momentum space:

$$T_E(k', k) = V(k', k) + \int d^3p V(k', p) \frac{1}{E - E(p) + i\epsilon} T_E(p, k) \qquad (1)$$

As is evident from the equation, nonlocal effective potentials may be directly employed (in fact these are proportional to the QBD T-matrices), unitarity is automatically restored, and nonrelativistic kinematics may be incorporated by use of the relativistic dispersion relation for $E(p)$. This is illustrated as the dotted line in Fig 4, where one sees that the effect of iterating the $\pi\pi$ potential is to weaken the scattering at higher invariant mass.

3.2   Annihilation to Hybrids

To this point attention has been restricted to 'exotic' channels such as I=2, where resonance contributions are forbidden. We now examine the issues involved in relaxing this constraint. Two possible intermediate states may be realized (at least at lowest order in the Fock space expansion), annihilation to intermediate mesons or annihilation to intermediate hybrid mesons. The latter process involves an intermediate state consisting of a quark – antiquark pair and a 'gluon' (where the gluon may be an excited flux tube or a constituent gluon, depending on one's picture of soft glue).

The nonrelativistic reduction of the one gluon exchange potential which describes the coupling of a $q\bar{q}$ pair to an intermediate perturbative gluon is[25]

$$V_{ann} = \frac{2\pi\alpha_s}{m^2} \left(\frac{3}{4} + \vec{S}_i \cdot \vec{S}_j \right) \delta(\vec{r}_{ij}) \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2}. \qquad (2)$$

However, the intermediate state in this case is a hybrid, which typically has a mass some 1000 MeV above low lying mesons with the same quark content. Thus one may expect that it is more realistic to employ a gluon propagator with a fictitious mass of roughly 1 GeV. One may incorporate this into the expression above by multiplying it by a factor f which is to be fit to the data and which we expect to be roughly -1. This rather speculative adjustment can be verified by comparing reactions with no annihilation to similar reactions where annihilation is permitted. For example, I=2 $\pi\pi$ scattering may be compared to I=0 $\pi\pi$; I=3/2 $K\pi$ scattering may be compared to I=1/2 $K\pi$; and K^+N scattering may be compared to πN scattering. All indicate that a negative value of f is required; a fit yields $f \approx -2.6$[25].

3.3 Coupled Channels

Even a cursory examination of typical hadronic scattering data reveals the importance of intermediate resonance states. Unfortunately the mechanism by which hadrons couple is poorly understood and surely involves complicated nonperturbative gluodynamics. The current best guess is the purely phenomenological $^3P_0$ model in which $q\bar{q}$ pairs are created with vacuum quantum numbers and combine with the parent quarks to produce the daughter mesons. Extensive calculations of meson and baryon decays have been made with moderate success (typical errors in the amplitude are 20% or less)[26].

Incorporating the $^3P_0$ model directly into the quark model is the most direct way to include the effects of intermediate resonances. This may be achieved most simply by writing the quark model in second quantized notation and including a $^3P_0$ term which creates and annihilates $q\bar{q}$ pairs. This is multiplied by a constant γ which may be determined by comparison to a specific channel (say, $\rho \to \pi\pi$).

$$
\begin{aligned}
\hat{H} &= \int dx\,\Big(-\frac{\nabla^2}{2m_q}b_x^\dagger b_x - \frac{\nabla^2}{2m_{\bar{q}}}d_x^\dagger d_x\Big) + \gamma \int dx\,(b_x^\dagger \sigma \cdot \overset{\leftrightarrow}{\nabla} d_x^\dagger + \mathrm{H.c.}). \\
&+ \frac{1}{2}\int dx\,dy\,(b_x^\dagger b_y^\dagger + d_x^\dagger d_y^\dagger)V(x-y)(b_y b_x + d_y d_x). \quad (3)
\end{aligned}
$$

The field theory is simplified by restricting the Fock space to the meson and meson-meson sectors of interest. Thus we project onto $|A\rangle$, $|BC\rangle$ by making the following Ansatz for the exact eigenstate:

$$
\begin{aligned}
|\Psi\rangle &= \int \varphi_A(r_1 - r_2)b_1^\dagger d_2^\dagger |0\rangle \\
&+ \int \sum_{BC} \Psi_{BC}(\frac{r_2 + r_4 - r_1 - r_3}{2})\varphi_B(r_1 - r_3)\varphi_C(r_2 - r_4)b_1^\dagger d_3^\dagger b_2^\dagger d_4^\dagger |0\rangle \ (4)
\end{aligned}
$$

Varying the reduced Hamiltonian matrix element with respect to the unknown meson φ_A and meson-meson Ψ_{BC} wavefunctions yields the coupled channel Schrödinger equation:

$$
\begin{aligned}
E\varphi_A(r) &= H_{q\bar{q}}(r)\varphi_A(r) \\
-\gamma \int \vec{\Sigma} \cdot (\nabla_B &+ \nabla_C + \nabla_{BC})\varphi_{0B}(r/2 - x)\varphi_{0C}(r/2 + x)\Psi_{BC}(-r/2), \quad (5) \\
\frac{-1}{2\mu_{13,24}}\nabla_R^2 &+ \int\int K_E(x,y,R)\Psi_{BC}(R') + \int\int V_E(x,y,R)\Psi_{BC}(R')
\end{aligned}
$$

$$- \quad 8\gamma \int \vec{\Sigma} \cdot (\nabla_B + \nabla_C + \nabla_{BC}) \varphi_{0B} \varphi_{0C} \varphi_A(-2R)$$

$$= \quad E\Psi_{BC}(R) + E \int N_E(x, y, R) \Psi_{BC}(R') \qquad (6)$$

Here r is the interquark radius in the meson channel and R is the intermeson distance in the meson-meson channel. Remarkably there is a simple relationship between these coordinates: $R = r/2$. K_E, V_E, and N_E represent the exchange kinetic energy, potential, and normalization kernels respectively. Wavefunctions with a 'naught' subscript φ_0 represent mesonic wavefunctions without the effects of channel mixing (so that we have assumed no wavefunction distortion in deriving this equation). The first of these equations is the nonrelativistic quark model ($H_{q\bar{q}}$) supplemented with a term which couples it to the meson-meson continuum – thereby 'unquenching' the quark model. The second equation is the resonating group equation which describes meson-meson scattering in the CQM (the Born order T-matrix for this equation is provided by the QBD). The term proportional to γ provides the desired coupling to intermediate resonances.

Eqns (5,6) may be solved with standard coupled channel methods and the effects of unquenching the quark model and of intermediate resonances in scattering problems may be examined. An example of this is given in Fig 5 where the effect of coupling virtual ρs to the $\pi\pi$ P-wave channel is studied. The ρ mass has been shifted down 80 MeV while the bare width is 10 MeV, this corresponds to an 'RPA' width of 90 MeV[27].
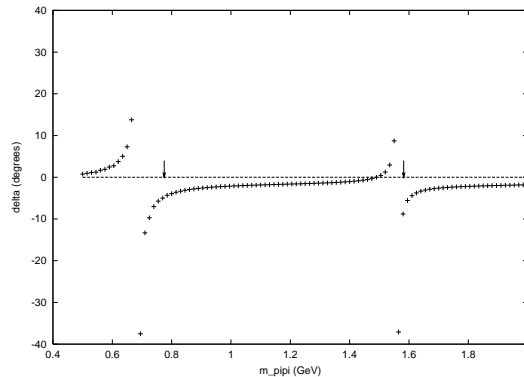

Fig. 5: I=1 $\pi\pi$ Scattering. The arrows indicate the locations of the 'bare' ρ and ρ' mesons.

3.4 Chiral Pions

The apparently successful description of $\pi\pi$ scattering evident in Fig 1 is perhaps surprising given the chiral nature of the pion and its interactions at low

energy. Explaining this success will go a long way towards explaining the unwarranted success of the CQM in the light quark sector[28]. One way to do this is to construct a model of strong QCD which incorporates the physics of chiral symmetry breaking at a microscopic level. In this way one may compute $\pi\pi$ scattering with composite particles while observing the dictates of chiral symmetry. It is possible to construct such a model by assuming a nontrivial QCD vacuum (typically a BCS-type vacuum) and building states on this vacuum with the random phase approximation to the full Bethe Salpeter equation. This has been done in Coulomb gauge QCD[29, 30], with similar calculations in the Schwinger-Dyson approach [31]

4   Conclusions

Studying and understanding hadronic interactions is vital to hadronic physics and is an important part of nuclear and electroweak physics. It is probable that a microscopic description of hadronic interactions is necessary if one wishes to understand these phenomena in extreme conditions or in poorly known channels. The constituent quark model provides an excellent starting point for developing the understanding required to construct a reliable model of strong QCD. In the meantime, it also serves as an excellent phenomenological guide to the interpretation of scattering experiments. The development of continuum field theoretic models capable of describing hadrons and their interactions is in its infancy – we look forward to their maturation and application to reaction processes.

References

[1] An example of such phases has just become known to us. Preliminary measurements by E852 indicate that the difference of the $\omega\pi$ system in S and D wave at the mass of the b_1 is -9 ± 3 degrees. This is to be compared to the prediction of -14 degrees[15].

[2] R. B. Wiringa, V. G. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995); S. Krewald, K. Nakayama, and J. Speth, Phys. Rept. **161**, 103 (1988);

B. Loiseau, "A review of the nucleon nucleon Paris potential," Given at *20th Brazilian Workshop on Nuclear Physics*, Guaratingueta, Sao Paulo, Brazil, 31 Aug - 4 Sep 1997; K. Holinde, Few Body Syst. Suppl. **8** (1995) 448.

[3] J.J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969); B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. **E6**, 515 (1997).

[4] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984); D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys. Lett. B **424**, 390 (1998).

[5] D. Richards, D. Sinclair, and D. Sivers, Phys. Rev. D42, 3191 (1990); A. Mihaly, H.R. Fiebig, H. Markum, and K. Rabitsch, Phys. Rev. D55, 3077 (1997); H.R. Fiebig, H. Markum, A. Mihaly, K. Rabitsch, and R.M. Woloshyn, Nucl. Phys. Proc. Suppl. 63, 188 (1998); C. Stewart and R. Koniuk, Phys. Rev. D57, 5581 (1998); C. Michael and P. Pennanen (UKQCD Collaboration), hep-lat/9901007 (Jan. 1999).

[6] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).

[7] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[8] M. Peskin, Nucl. Phys. **B156**, 365 (1979).

[9] D.A. Liberman, Phys. Rev. **D16**, 1542 (1977).

[10] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).

[11] C.S. Warke and R. Shanker, Phys. Rev. C**21**, 2643 (1980); M. Oka and K. Yazaki, Phys. Lett. **90B**, 41 (1980); C.S. Warke and R. Shanker, Phys. Rev. C **21**, 2643 (1980); J.E.T. Ribeiro, Z. Phys. **C5**, 27 (1980).

[12] T. Barnes and E. S. Swanson, Phys. Rev. D **46**, 131 (1992).

[13] E. S. Swanson, Annals Phys. **220**, 73 (1992).

[14] L.I. Schiff, *Quantum Mechanics* (McGraw Hill, New York, 1968).

[15] T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C **63**, 025204 (2001).

[16] T. Barnes, E. S. Swanson, and J. Weinstein, Phys. Rev. D **46**, 4868 (1992).

[17] T. Barnes and E. S. Swanson, Phys. Rev. C **49**, 1166 (1994).

[18] This may be traced to the importance of the quark-level spin-orbit interaction. Work is underway to test this.

[19] T. Barnes, S. Capstick, M. D. Kovarik, and E. S. Swanson, Phys. Rev. C **48**, 539 (1993).

[20] T. Barnes, N. Black, D. J. Dean, and E. S. Swanson, Phys. Rev. C **60**, 045202 (1999).

[21] C. Wong, E. S. Swanson, and T. Barnes, Phys. Rev. C **62**, 045201 (2000).

[22] K. Dooley, E. S. Swanson, and T. Barnes, Phys. Lett. B **275**, 478 (1992).

[23] J. F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev. D **38**, 2195 (1988).

[24] W. Hoogland *et al.*, Nucl. Phys. **B126**, 109 (1977); J.P. Prukop *et al.*, Phys. Rev. **D10**, 2055 (1974).

[25] Z. Li, M. Guidry, T. Barnes, and E. S. Swanson, hep-ph/9401326.

[26] L. Micu, Nucl. Phys. **B10**, 521 (1969); R. Carlitz and M. Kislinger, Phys. Rev. D **2**, 336 (1970); A. Le Yaouanc, L. Oliver, O. Pene, and J.-C. Raynal, Phys. Rev. D **8**, 2233 (1973); Phys. Lett. **71 B**, 397 (1977); *ibid* **72 B**, 57 (1977); R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987); W. Roberts and B. Silvestre-Brac, Few Body Syst. **11**, 171 (1992); P. Geiger and E. S. Swanson, Phys. Rev. D **50**, 6855 (1994).

[27] P. R. Page, E. S. Swanson, and A. P. Szczepaniak, Phys. Rev. D **59**, 034016 (1999).

[28] A. P. Szczepaniak and E. S. Swanson, hep-ph/0006306.

[29] A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **55**, 1578 (1997).

[30] J.R. Finger and J.E. Mandula, Nucl. Phys. **B199**, 168 (1982); A. Amer, A. LeYaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. Lett. **50**, 87 (1983); Phys. Rev. bf D28, 1530 (1983); S. Alder and A. Davis, Nucl. Phys. **B244**, 469 (1984).

[31] C. D. Roberts, R. T. Cahill, M. E. Sevior, and N. Iannella, Phys. Rev. D **49**, 125 (1994).

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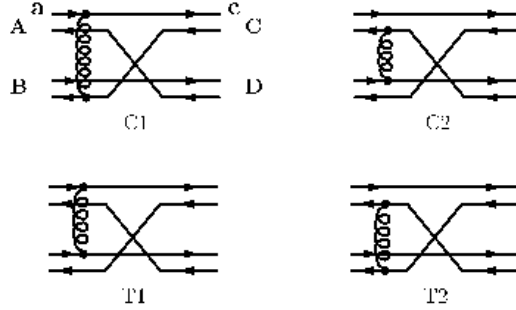

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A comparison of the predicted and experimental isotensor S-wave $\pi\pi$ phase shifts is shown in Fig 2. A sample cross section prediction relevant to charm suppression at RHIC is shown in Fig 3.
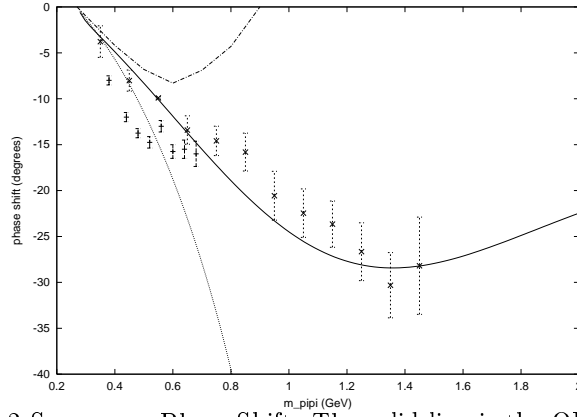

Fig. 2: The $I = 2$ S-wave $\pi\pi$ Phase Shift. The solid line is the QBD prediction[12], the dashed line is the one loop chiral prediction[23], while the lower dashed line is the tree order chiral result. Data are from [24].
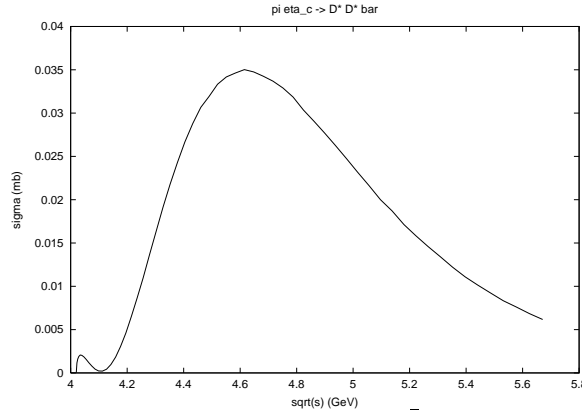

Fig. 3: Prediction of the $\pi\eta_c \to D^*\bar{D}^*$ Cross Section.

3 Limitations and Extensions of the CQM

As mentioned above, the CQM in general, and the QBD formalism in particular, suffers from several inadequacies. We summarize several of these and discuss methods for addressing them.

3.1 Unitarity, Relativistic Kinematics, and Nonlocality

Perhaps the simplest problem arises when the scattering is so strong that the tree level diagrams of the QBD method are inaccurate. This may be tested by comparing QBD predictions to more complete resonating group calculations (which iterate the scattering to all orders and can, in principle, include the effects of coupled channels, wavefunction distortion, etc). Such a comparison of $\rho\rho$ scattering in isospin 2 was made in Ref. [13] where the accuracy of the QBD results were explicitly demonstrated. Nevertheless, it is worth observing that strong interactions may be accounted for by extracting effective potentials for the process in question and iterating the potential in the appropriate Schrödinger equation. An example of this is shown in Fig 4, where the effective $\pi\pi$ interaction has been extracted from the QBD $\pi\pi$ T-matrix and iterated in the nonrelativistic Schrödinger equation. This is shown as a dashed line. Evidently, the agreement with data is ruined by this procedure (I am presenting the worst possible case – the procedure works very well in general). This is because the light mass of the pions (and the large invariant masses at which the formalism is being applied) requires relativistic kinematics to be employed (this has been used in the QBD prediction of Fig 2). Thus the nonrelativistic Schrödinger equation is inappropriate for iterating the effective $\pi\pi$ interaction.
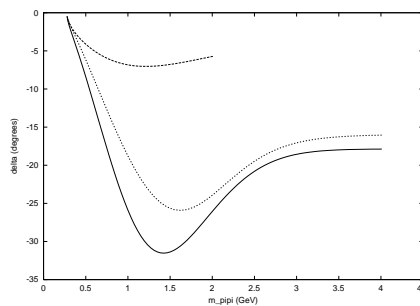

Fig. 4: I=2 $\pi\pi$ Scattering. The solid line is the QBD prediction, the dashed line is the nonrelativistic local prediction, while the dotted line is the nonlocal, relativistic prediction.

A related problem is the nonlocality of the $\pi\pi$ T-matrix (because composite particles are being scattered). Any extracted effective potential which is local

will induce unknown errors after being iterated. Fortunately, all three problems may be dealt with by choosing to solve for the T-matrix directly in momentum space:

$$T_E(k', k) = V(k', k) + \int d^3p V(k', p) \frac{1}{E - E(p) + i\epsilon} T_E(p, k) \tag{1}$$

As is evident from the equation, nonlocal effective potentials may be directly employed (in fact these are proportional to the QBD T-matrices), unitarity is automatically restored, and nonrelativistic kinematics may be incorporated by use of the relativistic dispersion relation for $E(p)$. This is illustrated as the dotted line in Fig 4, where one sees that the effect of iterating the $\pi\pi$ potential is to weaken the scattering at higher invariant mass.

3.2 Annihilation to Hybrids

To this point attention has been restricted to 'exotic' channels such as I=2, where resonance contributions are forbidden. We now examine the issues involved in relaxing this constraint. Two possible intermediate states may be realized (at least at lowest order in the Fock space expansion), annihilation to intermediate mesons or annihilation to intermediate hybrid mesons. The latter process involves an intermediate state consisting of a quark – antiquark pair and a 'gluon' (where the gluon may be an excited flux tube or a constituent gluon, depending on one's picture of soft glue).

The nonrelativistic reduction of the one gluon exchange potential which describes the coupling of a $q\bar{q}$ pair to an intermediate perturbative gluon is[25]

$$V_{ann} = \frac{2\pi\alpha_s}{m^2} \left(\frac{3}{4} + \vec{S}_i \cdot \vec{S}_j \right) \delta(\vec{r}_{ij}) \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2}. \tag{2}$$

However, the intermediate state in this case is a hybrid, which typically has a mass some 1000 MeV above low lying mesons with the same quark content. Thus one may expect that it is more realistic to employ a gluon propagator with a fictitious mass of roughly 1 GeV. One may incorporate this into the expression above by multiplying it by a factor f which is to be fit to the data and which we expect to be roughly -1. This rather speculative adjustment can be verified by comparing reactions with no annihilation to similar reactions where annihilation is permitted. For example, I=2 $\pi\pi$ scattering may be compared to I=0 $\pi\pi$; I=3/2 $K\pi$ scattering may be compared to I=1/2 $K\pi$; and K^+N scattering may be compared to πN scattering. All indicate that a negative value of f is required; a fit yields $f \approx -2.6$[25].

3.3   Coupled Channels

Even a cursory examination of typical hadronic scattering data reveals the importance of intermediate resonance states. Unfortunately the mechanism by which hadrons couple is poorly understood and surely involves complicated nonperturbative gluodynamics. The current best guess is the purely phenomenological $^3P_0$ model in which $q\bar{q}$ pairs are created with vacuum quantum numbers and combine with the parent quarks to produce the daughter mesons. Extensive calculations of meson and baryon decays have been made with moderate success (typical errors in the amplitude are 20% or less)[26].

Incorporating the $^3P_0$ model directly into the quark model is the most direct way to include the effects of intermediate resonances. This may be achieved most simply by writing the quark model in second quantized notation and including a $^3P_0$ term which creates and annihilates $q\bar{q}$ pairs. This is multiplied by a constant γ which may be determined by comparison to a specific channel (say, $\rho \to \pi\pi$).

$$
\begin{aligned}
\hat{H} &= \int dx\,\Big(-\frac{\nabla^2}{2m_q}b_x^\dagger b_x - \frac{\nabla^2}{2m_{\bar{q}}}d_x^\dagger d_x\Big) + \gamma\int dx\,(b_x^\dagger \sigma\cdot\overset{\leftrightarrow}{\nabla}d_x^\dagger + \text{H.c.}). \\
&+ \frac{1}{2}\int dx\,dy\,(b_x^\dagger b_y^\dagger + d_x^\dagger d_y^\dagger)V(x-y)(b_y b_x + d_y d_x).
\end{aligned}
\tag{3}
$$

The field theory is simplified by restricting the Fock space to the meson and meson-meson sectors of interest. Thus we project onto $|A\rangle$, $|BC\rangle$ by making the following Ansatz for the exact eigenstate:

$$
\begin{aligned}
|\Psi\rangle &= \int \varphi_A(r_1 - r_2)b_1^\dagger d_2^\dagger|0\rangle \\
&+ \int\sum_{BC}\Psi_{BC}\Big(\frac{r_2+r_4-r_1-r_3}{2}\Big)\varphi_B(r_1-r_3)\varphi_C(r_2-r_4)b_1^\dagger d_3^\dagger b_2^\dagger d_4^\dagger|0\rangle
\end{aligned}
\tag{4}
$$

Varying the reduced Hamiltonian matrix element with respect to the unknown meson φ_A and meson-meson Ψ_{BC} wavefunctions yields the coupled channel Schrödinger equation:

$$
\begin{aligned}
E\varphi_A(r) &= H_{q\bar{q}}(r)\varphi_A(r) \\
-\gamma\int\vec{\Sigma}\cdot(\nabla_B &+ \nabla_C + \nabla_{BC})\varphi_{0B}(r/2-x)\varphi_{0C}(r/2+x)\Psi_{BC}(-r/2), \tag{5}\\
\frac{-1}{2\mu_{13,24}}\nabla_R^2 &+ \int\int K_E(x,y,R)\Psi_{BC}(R') + \int\int V_E(x,y,R)\Psi_{BC}(R')
\end{aligned}
$$

$$- \quad 8\gamma \int \vec{\Sigma} \cdot (\nabla_B + \nabla_C + \nabla_{BC})\varphi_{0B}\varphi_{0C}\varphi_A(-2R)$$

$$= \quad E\Psi_{BC}(R) + E \int N_E(x, y, R)\Psi_{BC}(R') \tag{6}$$

Here r is the interquark radius in the meson channel and R is the intermeson distance in the meson-meson channel. Remarkably there is a simple relationship between these coordinates: $R = r/2$. K_E, V_E, and N_E represent the exchange kinetic energy, potential, and normalization kernels respectively. Wavefunctions with a 'naught' subscript φ_0 represent mesonic wavefunctions without the effects of channel mixing (so that we have assumed no wavefunction distortion in deriving this equation). The first of these equations is the nonrelativistic quark model ($H_{q\bar{q}}$) supplemented with a term which couples it to the meson-meson continuum – thereby 'unquenching' the quark model. The second equation is the resonating group equation which describes meson-meson scattering in the CQM (the Born order T-matrix for this equation is provided by the QBD). The term proportional to γ provides the desired coupling to intermediate resonances.

Eqns (5,6) may be solved with standard coupled channel methods and the effects of unquenching the quark model and of intermediate resonances in scattering problems may be examined. An example of this is given in Fig 5 where the effect of coupling virtual ρs to the $\pi\pi$ P-wave channel is studied. The ρ mass has been shifted down 80 MeV while the bare width is 10 MeV, this corresponds to an 'RPA' width of 90 MeV[27].
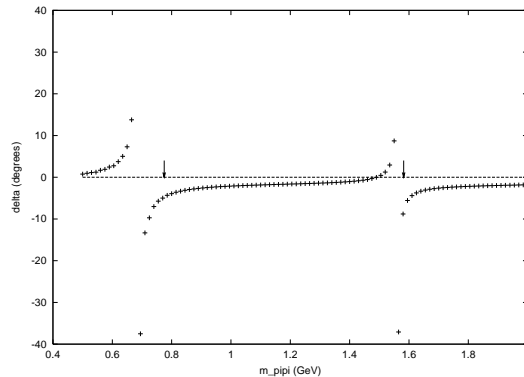

Fig. 5: I=1 $\pi\pi$ Scattering. The arrows indicate the locations of the 'bare' ρ and ρ' mesons.

3.4 Chiral Pions

The apparently successful description of $\pi\pi$ scattering evident in Fig 1 is perhaps surprising given the chiral nature of the pion and its interactions at low

energy. Explaining this success will go a long way towards explaining the unwarranted success of the CQM in the light quark sector[28]. One way to do this is to construct a model of strong QCD which incorporates the physics of chiral symmetry breaking at a microscopic level. In this way one may compute $\pi\pi$ scattering with composite particles while observing the dictates of chiral symmetry. It is possible to construct such a model by assuming a nontrivial QCD vacuum (typically a BCS-type vacuum) and building states on this vacuum with the random phase approximation to the full Bethe Salpeter equation. This has been done in Coulomb gauge QCD[29, 30], with similar calculations in the Schwinger-Dyson approach [31]

4 Conclusions

Studying and understanding hadronic interactions is vital to hadronic physics and is an important part of nuclear and electroweak physics. It is probable that a microscopic description of hadronic interactions is necessary if one wishes to understand these phenomena in extreme conditions or in poorly known channels. The constituent quark model provides an excellent starting point for developing the understanding required to construct a reliable model of strong QCD. In the meantime, it also serves as an excellent phenomenological guide to the interpretation of scattering experiments. The development of continuum field theoretic models capable of describing hadrons and their interactions is in its infancy – we look forward to their maturation and application to reaction processes.

References

[1] An example of such phases has just become known to us. Preliminary measurements by E852 indicate that the difference of the $\omega\pi$ system in S and D wave at the mass of the b_1 is -9 ± 3 degrees. This is to be compared to the prediction of -14 degrees[15].

[2] R. B. Wiringa, V. G. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995); S. Krewald, K. Nakayama, and J. Speth, Phys. Rept. **161**, 103 (1988);

B. Loiseau, "A review of the nucleon nucleon Paris potential," Given at *20th Brazilian Workshop on Nuclear Physics*, Guaratingueta, Sao Paulo, Brazil, 31 Aug - 4 Sep 1997; K. Holinde, Few Body Syst. Suppl. **8** (1995) 448.

[3] J.J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969); B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. **E6**, 515 (1997).

[4] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984); D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys. Lett. B **424**, 390 (1998).

[5] D. Richards, D. Sinclair, and D. Sivers, Phys. Rev. D42, 3191 (1990); A. Mihaly, H.R. Fiebig, H. Markum, and K. Rabitsch, Phys. Rev. D55, 3077 (1997); H.R. Fiebig, H. Markum, A. Mihaly, K. Rabitsch, and R.M. Woloshyn, Nucl. Phys. Proc. Suppl. 63, 188 (1998); C. Stewart and R. Koniuk, Phys. Rev. D57, 5581 (1998); C. Michael and P. Pennanen (UKQCD Collaboration), hep-lat/9901007 (Jan. 1999).

[6] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994).

[7] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[8] M. Peskin, Nucl. Phys. **B156**, 365 (1979).

[9] D.A. Liberman, Phys. Rev. **D16**, 1542 (1977).

[10] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).

[11] C.S. Warke and R. Shanker, Phys. Rev. C**21**, 2643 (1980); M. Oka and K. Yazaki, Phys. Lett. **90B**, 41 (1980); C.S. Warke and R. Shanker, Phys. Rev. C **21**, 2643 (1980); J.E.T. Ribeiro, Z. Phys. **C5**, 27 (1980).

[12] T. Barnes and E. S. Swanson, Phys. Rev. D **46**, 131 (1992).

[13] E. S. Swanson, Annals Phys. **220**, 73 (1992).

[14] L.I. Schiff, *Quantum Mechanics* (McGraw Hill, New York, 1968).

[15] T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C **63**, 025204 (2001).

[16] T. Barnes, E. S. Swanson, and J. Weinstein, Phys. Rev. D **46**, 4868 (1992).

[17] T. Barnes and E. S. Swanson, Phys. Rev. C **49**, 1166 (1994).

[18] This may be traced to the importance of the quark-level spin-orbit interaction. Work is underway to test this.

[19] T. Barnes, S. Capstick, M. D. Kovarik, and E. S. Swanson, Phys. Rev. C **48**, 539 (1993).

[20] T. Barnes, N. Black, D. J. Dean, and E. S. Swanson, Phys. Rev. C **60**, 045202 (1999).

[21] C. Wong, E. S. Swanson, and T. Barnes, Phys. Rev. C **62**, 045201 (2000).

[22] K. Dooley, E. S. Swanson, and T. Barnes, Phys. Lett. B **275**, 478 (1992).

[23] J. F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev. D **38**, 2195 (1988).

[24] W. Hoogland *et al.*, Nucl. Phys. **B126**, 109 (1977); J.P. Prukop *et al.*, Phys. Rev. **D10**, 2055 (1974).

[25] Z. Li, M. Guidry, T. Barnes, and E. S. Swanson, hep-ph/9401326.

[26] L. Micu, Nucl. Phys. **B10**, 521 (1969); R. Carlitz and M. Kislinger, Phys. Rev. D **2**, 336 (1970); A. Le Yaouanc, L. Oliver, O. Pene, and J.-C. Raynal, Phys. Rev. D **8**, 2233 (1973); Phys. Lett. **71 B**, 397 (1977); *ibid* **72 B**, 57 (1977); R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987); W. Roberts and B. Silvestre-Brac, Few Body Syst. **11**, 171 (1992); P. Geiger and E. S. Swanson, Phys. Rev. D **50**, 6855 (1994).

[27] P. R. Page, E. S. Swanson, and A. P. Szczepaniak, Phys. Rev. D **59**, 034016 (1999).

[28] A. P. Szczepaniak and E. S. Swanson, hep-ph/0006306.

[29] A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **55**, 1578 (1997).

[30] J.R. Finger and J.E. Mandula, Nucl. Phys. **B199**, 168 (1982); A. Amer, A. LeYaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. Lett. **50**, 87 (1983); Phys. Rev. bf D28, 1530 (1983); S. Alder and A. Davis, Nucl. Phys. **B244**, 469 (1984).

[31] C. D. Roberts, R. T. Cahill, M. E. Sevior, and N. Iannella, Phys. Rev. D **49**, 125 (1994).